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The Eightfold Way to Color Geometrodynamics

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Color models of strong interactions are generalized to a $GL(8,\mathbb{C})^f \otimes GL(8,\mathbb{C})^c$ gauge theory incorporating space-time curvature and Cartan's torsion. Following Salam, the dynamics is determined by an Einstein–Dirac-type Lagrangian. The resulting field equations are a *nonlinear* (due to the torsion) Heisenberg–Pauli–Weyl equation for the fundamental spinor fields and a generalized Einstein equation for the background metric of hadronic dimensions. According to this model baryonic quarks are confined in *geon* (black soliton)-type objects by the tensor gluons of *strong* gravity. This approach also leads to a black soliton mass formula which is in qualitative agreement with part of the baryon spectrum. Hadronic mesons are interpreted as gluon strings trapped in a multiconnected space-time. Interrelations of color geometrodynamics with other "bag" models are pointed out. Finally, the conceptual origin of this space-time foundation of quark confinement is presented.

1. INTRODUCTION

The formulation of Dirac's theory of the electron in the frame of general relativity has to its credit one feature which should be appreciated even by the atomic physicist who feels safe in ignoring the role of gravitation in the building-up of the elementary particles: its explanation of the quantum mechanical principle of "gauge invariance" that connects Dirac's ψ with the electromagnetic potentials.

This view put forth in 1950 by Herman Weyl is revived in *color geometrodynamics* (CGMD): Matter is represented by $f \times c$ fundamental spinor fields $\psi^{(f,c)}$ (distinguished by f flavor and c color degrees of freedom) which are coupled to a Lagrangian invariantly constructed from the gauge potentials of strong interactions. However, unlike *quantum chromodynamics* (Gell-Mann et al., 1978), which assumes $U(f) \otimes U(c)$ as "gauge group," following Weyl, the tensor forces of *strong gravity* (Isham et al., 1971) should play an equivalently important role for a description of

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strong interactions. Consequently $GL(2f,\mathbb{C})\otimes GL(2c,\mathbb{C})$ is taken as the gauge group of CGMD whereas its dynamics is determined by a gauge-invariant generalization of the Einstein–Hilbert action together with a Dirac Lagrangian generalized to a curved space-time of hadronic dimension (Salam, 1973). The latter is characterized by the modified *Planck length*

$$l^* \equiv \left(8\pi\hbar G_S/c^3\right)^{1/2} = (8\pi)^{1/2}\hbar/cM^* \tag{1.1}$$

or the *Planck mass* $M^* \sim 1$ GeV of strong gravity.

As is well known from general relativity with spin and torsion (Hehl et al., 1976) Cartan's notion of *torsion* (Cartan, 1922, 1923–25) of the underlying space-time induces nonlinear spinor terms into the Dirac equation. In the generalization considered here the resulting *Heisenberg–Pauli–Weyl spinor equation* (Weyl, 1950) gives rise to a *nonlinear coupling* also among the *different* fundamental spinor fields, similarly as in *Heisenberg's unified field theory of elementary particles* (Heisenberg, 1966; 1974). Based on this geometrodynamical gauge model an unconventional mechanism of quark confinement has recently been proposed (Mielke, 1977d).

In this paper further speculations are offered with the aim of incorporating all known interactions between particles into a unifying scheme on a *semiclassical* level. To be more specific, the *colored* quark hypothesis (see Greenberg and Nelson 1977, for a review) will be combined with the two-tensor theory of gravitation (Isham et al., 1971, 1973, 1974; see also Sivaram and Sinha, 1979, for a recent review). Including *charmed* quarks and regarding the lepton number as the fourth color (Pati and Salam, 1974) a $GL(8,\mathbb{C})^f\otimes GL(8,\mathbb{C})^c$ gauge unification of all basic particle forces may emerge. As indicated by the title of this paper, this model may be regarded as a geometrical extension of the celebrated "*eightfold way*" scheme of Gell-Mann and Ne'eman (1964).

In Section 2 the geometrical foundation (Mielke, 1979a) of CGMD based on the theory of *fiber bundles* is presented. Its dynamics is defined by a gauge-invariant Einstein–Dirac-type Lagrangian modeled on a curved space-time of hadronic dimensions.

As a result the 4^2-fold set of fundamental spinor fields is governed by the *nonlinear* spinor equation of the Heisenberg–Pauli–Weyl-type already mentioned above, whereas the tensor dominated contribution of strong interactions then has to satisfy generalized Einstein field equations. Under the assumption that the matter is described by *localized* solutions of the spinor equation, the strong gravity metric tends to the vacuum solutions of Einstein's field equations far away from the center of these "*solitonlike*" objects. Then the asymptotical background is given by a Kerr–Newman-type metric in the stationary case. Unlike general relativity this exterior geometry can be distinguished, besides by mass, angular momentum, and charge, by its color and flavor content, as will be discussed in Section 3.

In Section 4 tentative particle assignments are made according to the CGMD scheme. Since a color symmetry with an exact $U(4)^c$ subgroup would be too large physically, it is tempting to assume it to be broken in such a way that leptons remain (approximately) massless whereas quarks pick up real masses similarly as in Goldstone's model theory (see Taylor 1976 for a review). Therefore the nonlinear interactions between the leptons can be almost neglected as in conventional theories. This has to be contrasted to the massive case where the nonlinear spinor terms are expected to give rise to *bound states* of quarks. In a related, but simplified, model (Deppert and Mielke, 1979) with *n scalar* "quarks" obeying a nonlinear Heisenberg–Klein–Gordon equation, such a binding of the scalar fields to localized spherical waves has been found. The latter effect may be further enhanced by *confining curvature potentials* self-consistently generated in CGMD by Einstein-type equations. This expectation is backed up by the self-attracting feature of these *tensor-gluons*, a property of CGMD which it does not share with Yang–Mills-type gauge theories (Coleman and Smarr, 1977). Accordingly, if baryons are *extended* objects, they should be described by *black solitons* (Salam and Strathdee, 1976) which strongly resemble Wheeler's prior construction of *geons* [= gravitational–electromagnetic entities (Wheeler, 1962)]. If these objects collide, topological bridges should occur with the flux lines of the color and flavor gauge fields locked in. The "mouths" of these *wormholes* would give the impression of a quark–antiquark bound state usually postulated for strongly interacting mesons.

In Section 5, connections of CGMD with phenomenological *bag models* of the MIT and SLAC groups are indicated on the classical level.

Section 6 touches upon the fundamental issue of to what extent the nonexistence of free quarks rests upon the structure of space-time itself.

2. THE GEOMETRICAL GAUGE MODEL

The generalization to be tackled—namely, the generalization of the $SL(6,\mathbb{C})$ gauge theory of strong interactions (Isham et al., 1973) to one with an additional charm and four (hidden) color degrees of freedom— is formally a straightforward task. To be more precise, a *principal fiber bundle* (Kobayashi and Nomizu, 1963) over a pseudo-Riemannian space-time M^4

with Lorentzian signature $(1,-1,-1,-1)$ and the structure group

$$G \equiv GL(8,\mathbb{C})^f \otimes GL(8,\mathbb{C})^c \supset U(4)_L^f \otimes U(4)_R^f \otimes U(4)_L^c \otimes U(4)_R^c \quad (2.1)$$

will be considered, where f and c denote the flavor and color degrees of freedom (respectively). L and R correspond to left and right helicities of the fermions. The bundle of linear frames $L(M^4)$ is locally given by

$$L_\mu \equiv \tfrac{1}{2}\{L_{\mu a}^{(f)j}\gamma^a + i^* L_{\mu a}^{(f)j}\gamma^a\gamma^5\}\lambda_j^{(f)} \oplus \tfrac{1}{2}\{L_{\mu a}^{(c)j}\gamma^a + i^* L_{\mu a}^{(c)j}\gamma^a\gamma^5\}\lambda_j^{(c)} \quad (2.2)$$

[The conventions for the Dirac matrices are as in Bjorken and Drell (1964), whereas the n^2 vector operators of $U(n)$ are represented by the generalized Gell-Mann matrices λ_j normalized to $\mathrm{Tr}(\lambda_i\lambda_j)=2\delta_{ij}$.]

The G bundle $L(M^4)$ (=set of all 4^4-bein fields in space-time) possesses the gauge-invariant fiber metric

$$f_{\mu\nu} \equiv \tfrac{1}{2}(f_{\mu\nu}^{(f)} \oplus f_{\mu\nu}^{(c)}) \equiv \tfrac{1}{4}\mathrm{Tr}(L_\mu L_\nu) \quad (2.3)$$

along with the gravitational metric $g_{\mu\nu}$ (which corresponds to the gauge group $SL(2,\mathbb{C})$ of conventional general relativity).

The linear connection

$$B_\mu \equiv \tfrac{1}{4}\Lambda_{\mu ab}^{(f)j}\sigma^{ab}\lambda_j^{(f)} \oplus \tfrac{1}{4}\Lambda_{\mu ab}^{(c)j}\sigma^{ab}\lambda_j^{(c)} \oplus A_\mu \quad (2.4)$$

can, as usual, be expanded in terms of the 4^4 infinitesimal Hermitian generators of the noncompact group G. Because of later importance the terms corresponding to the unitary subgroups have been listed separately by

$$A_\mu \equiv \tfrac{1}{2}\{A_\mu^{(f)j} + {}^*A_\mu^{(f)j}\gamma^5\}\lambda_j^{(f)} \oplus \tfrac{1}{2}\{A_\mu^{(c)j} + {}^*A_\mu^{(c)j}\gamma^5\}\lambda_j^{(c)} \quad (2.5)$$

With respect to B_μ a gauge-covariant differentiation

$$D_\mu\psi \equiv \partial_\mu\psi + iB_\mu\psi \quad (2.6)$$

is defined in the local cross section

$$\psi \equiv \{\psi^{(f,c)} \mid f,c=1,\ldots,4\} \quad (2.7)$$

of the bundle of 4^2 Dirac spinors *associated* with $L(M^4)$.

In terms of the connection 1-form

$$B = B_\mu\, dx^\mu \quad (2.8)$$

the torsion 2-form

$$T=\tfrac{1}{2}L_\alpha T^\alpha{}_{\mu\nu}\,dx^\mu\wedge dx^\nu \tag{2.9}$$

is defined by

$$T\equiv DL=dL+i[B,L] \tag{2.10}$$

i.e., via the *first structure equation of É. Cartan.* It satisfies the *first Bianchi identity*

$$DT=i[C,L] \tag{2.11}$$

The corresponding curvature 2-form ["curvature operator," Misner et al., 1973 (cited hereafter as MTW), p. 365]

$$\begin{aligned} C&=\tfrac{1}{2}C_{\mu\nu}\,dx^\mu\wedge dx^\nu \\ &=\tfrac{1}{4}L_\alpha\wedge L_\beta R^{\alpha\beta}{}_{\mu\nu}\,dx^\mu\wedge dx^\nu \end{aligned} \tag{2.12}$$

is given by the second *structure equation* of *É. Cartan* (Kobayashi and Nomizu, 1963, p. 78)

$$C=dB+iB\wedge B \tag{2.13}$$

and satisfies the *second Bianchi identity*

$$DC=0 \tag{2.14}$$

In gauge theory (2.13) is more familiar in the local form

$$C_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu+i[B_\mu,B_\nu] \tag{2.15}$$

Likewise, the field strengths F corresponding to the unitary subgroup of G locally read

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+i[A_\mu,A_\nu] \tag{2.16}$$

For later purposes the tensor $^*F$ *dual* to F is introduced via:

$$^*F_{\mu\nu}\equiv\tfrac{1}{2}|f|^{1/2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta} \tag{2.17}$$

In order to link the internal gauge symmetry to the curved space-time the "metric" condition

$$\nabla^B_\mu L^\alpha \equiv \partial_\mu L^\alpha + i\left[B_\mu, L^\alpha\right] + \Lambda^\alpha{}_{\beta\mu} L^\beta = 0 \tag{2.18}$$

will be imposed on the covariant derivative ∇^B acting on the bundle $L(M^4)$. Then, geometric objects can be defined that are invariant not only with respect to the group $\mathcal{G}$ of *local* gauge transformations but also with respect to the diffeomorphism group $\mathcal{D}$ of general coordinate transformations (Isham et al., 1973).

This is the case for the geometrodynamical Lagrangian density

$$\begin{aligned}\mathfrak{L}_{\mathrm{GMD}} = \frac{\hbar c}{2l^{*2}} \mathfrak{L}_W + \mathfrak{L}_D = \frac{\hbar c}{2l^{*2}} |f|^{1/2} \Big\{ & i\operatorname{Tr}(C_{\mu\nu}[L^\mu, L^\nu]) - 2\Lambda \\ & + l^{*2}\Big[i\bar{\psi} L^\mu D_\mu \psi - i(\overline{D_\mu \psi}) L^\mu \psi - 2\Big(\frac{\mu c}{\hbar}\Big)\bar{\psi}\psi \Big]\Big\}\end{aligned} \tag{2.19}$$

which defines the basic model. It will be referred to as (classical) *color geometrodynamics* (CGMD), since it is known that a complete *Rainich geometrization* of the fermion fields is in principle possible [at least in the case $G = GL(2, C)$], Kuchař, 1965). The modified *Planck length* (1.1) (or the *Planck mass* $M^* \sim 1$ GeV of *strong gravity* (Isham et al., 1971)) is the coupling constant of this model. It can be shown that $\mathfrak{L}_{\mathrm{GMD}}$ in the absence of spinor fields reduces to the familiar Einstein Lagrangian density

$$\mathfrak{L}_{\mathrm{GMD}}(\psi = 0) = \mathfrak{L}_E \equiv \frac{\hbar c}{2l^{*2}} |f|^{1/2} \{R(f) - 2\Lambda\} \tag{2.20}$$

The field equations can be obtained from the geometrodynamical Lagrangian by familiar variational principles. As the details of these derivations have been presented elsewhere (Mielke, 1979a), it is enough to collect here the main results.

(a) Varying $\mathfrak{L}_{\mathrm{GMD}}$ for the 1-form L of the linear frame bundle yields the *Einstein-type field equations*

$$G_{\mu\nu} + \Lambda f_{\mu\nu} = \frac{i}{2} l^{*2} \{ \bar{\psi} L_\mu D_\nu \psi - \overline{D_\nu \psi}\, L_\mu \psi \} \tag{2.21}$$

with *cosmological term.*

(b) Varying for $\delta \mathfrak{L}_{\mathrm{GMD}} / \delta\bar{\psi}$ the spinor equation

$$i\Big\{ L^\mu D_\mu + \frac{i}{2}\left[B_\mu, L^\mu\right] + \frac{1}{2}\partial_\mu L^\mu + \frac{i\mu c}{\hbar} \Big\}\psi = 0 \tag{2.22}$$

will be obtained.

(c) By varying $\mathcal{L}_{\mathrm{GMD}}$ for the contorsional part (Hehl and Datta, 1971) of the gauge connection B_μ the axial vector of the spin-unitary-spin current of the fundamental spinor fields is related to Cartan's torsion tensor (2.9) as is the case of conventional general relativity with spin and torsion (Hehl et al., 1976).

If this torsional relation is substituted into (2.22) the G-gauge-invariant generalization

$$\boxed{\left\{ iL^\mu \nabla_\mu - \frac{3}{8} l^{*2} \bar{\psi} L^5 L_\mu \psi L^5 L^\mu - \frac{\mu c}{\hbar} \right\} \psi = 0}, \tag{2.23}$$

$$L^5 \equiv \frac{i}{4!} |f|^{-1/2} \epsilon^{\alpha\beta\gamma\delta} L_\alpha \wedge L_\beta \wedge L_\gamma \wedge L_\delta$$

of the *nonlinear spinor equation*

$$\left\{ i\gamma^\mu \partial_\mu + \frac{\epsilon}{2} l^2 \left[\gamma_\mu \bar{\psi} \gamma^\mu \psi + \gamma_\mu \gamma^5 (\bar{\psi} \gamma^\mu \gamma^5 \psi) \right] - \frac{\mu c}{\hbar} \right\} \psi = 0 \tag{2.24}$$

proposed 1958 by Heisenberg and Pauli (Heisenberg, 1966, 1974) is the result. In a $SL(2,C)$ gauge theory of gravitation, Weyl derived already 1950 a similar equation with a self-interaction of the axial-vector type. The first term in (2.23) generalizes the Dirac operator to the curved space-time (see Schrödinger, 1932) of strong gravity. The second term due to torsion generates a more general self-coupling of the spinors compared to the Heisenberg–Pauli equation. The latter was originally devised to be invariant only with respect to the group $U(2)$ of isotopic spin, which incorporates only "*isotorsion*" (see also Finkelstein, 1961). Since the parity-invariant form has been invoked in (2.24), contrary to the original, dilatation-invariant formulation (Heisenberg, 1966), a mass term may be retained.

3. COLORED AND FLAVORED KERR–NEWMAN HOLES

In a hierarchical symmetry-breaking scheme it is physically more realistic to assume that the $GL(8,\mathbb{C})^f \otimes GL(8,\mathbb{C})^c$ gauge symmetry of L_{GMD} is broken down to $U(4)^f_L \otimes U(4)^f_R \otimes U(4)^c_L \otimes U(4)^c_R$ by the presence of the $\mathfrak{D}$-invariant Yang–Mills Lagrangian (Yang and Mills, 1954)

$$\mathcal{L}_{\mathrm{Y\text{-}M}} \equiv -\frac{\hbar c}{2\alpha} |f|^{1/2} \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu}) \tag{3.1}$$

generalized to curved space-time (see, e.g., Charap and Duff, 1977).

$$\alpha \equiv g^2/\hbar c \tag{3.2}$$

denotes a dimensionless coupling constant.

Disregarding for the moment the fundamental spinor fields ψ the complete Lagrangian density

$$\mathcal{L} = \mathcal{L}_{\text{GMD}} + \mathcal{L}_{\text{Y-M}} \tag{3.3}$$

yields, upon variation for $\delta\mathcal{L}/\delta f^{\mu\nu}$, the Einstein–Maxwell-type equations of CGMD:

$$R_{\mu\nu} - \frac{1}{2} Rf_{\mu\nu} + \Lambda f_{\mu\nu} = \frac{l^{*2}}{\alpha} \operatorname{Tr}\left(F_{\mu}{}^{\rho}F_{\nu\rho} + {}^{*}F_{\mu}{}^{\rho *}F_{\nu\rho}\right) \tag{3.4}$$

The field strengths F coupled to an external source S have to satisfy the *Yang–Mills equations*

$$DF \equiv dF + i[A, F] = 0, \qquad D^{*}F = \alpha^{*}S \tag{3.5}$$

which in a *curved* space-time locally read (MTW, p. 81)

$$\nabla^{\mu *}F_{\mu\nu} + i[A^{\mu}, {}^{*}F_{\mu\nu}] = 0, \qquad \nabla^{\mu}F_{\mu\nu} + i[A^{\mu}, F_{\mu\nu}] = \alpha S_{\nu} \tag{3.6}$$

From (3.6) it can be inferred that the *current* defined by

$$j_{\nu} \equiv \alpha S_{\nu} + \Lambda_{\mu\rho}{}^{\mu}F^{\rho}{}_{\nu} + \Lambda_{\mu\rho\nu}F^{\mu\rho} - i[A^{\mu}, F_{\mu\nu}] \tag{3.7}$$

is *locally* conserved

$$\partial^{\nu} j_{\nu} = 0 \tag{3.8}$$

Provided the charge density $|f|^{1/2} j^{0}$ is sufficiently *localized*, with respect to the stationary background (3.11) equations (3.6) admit the matrix-valued 1-form

$$A = -\left(\frac{\alpha}{8\pi}\right)^{1/2} l^{*} \frac{Qr}{\bar{\rho}^{2}} \left(c\, dt - J_{3} \frac{\hbar}{Mc} \sin^{2}\theta\, d\phi\right) \tag{3.9}$$

as a solution. This is in complete analogy to the Abelian case (MTW, p. 898), except that the generalized charge operator

$$Q = \int_{\text{spacelike hypersurface}} |f|^{1/2} j^{\nu}\, d\Sigma_{\nu} \tag{3.10}$$

accounts not only for the color (Perry, 1977) but also for the flavor degrees of freedom of the strong background metric. Then, an exact solution of

(3.4) is of the Kerr–Newman–de Sitter-type (Carter, 1973), which for $\Lambda=0$ (MTW, p. 877) reads

$$ds^2=f_{\mu\nu}\,dx^\mu\,dx^\nu=\frac{\Delta}{\bar{\rho}^2}\left[c\,dt-J_3\frac{\hbar}{Mc}\sin^2\theta\,d\phi\right]^2$$

$$-\frac{1}{\bar{\rho}^2}\sin^2\theta\left\{\left[r^2+\frac{J(J+1)\hbar^2}{M^2c^2}\right]d\phi-J_3\frac{\hbar}{M}\,dt\right\}^2-\frac{\bar{\rho}^2}{\Delta}\,dr^2-\bar{\rho}^2\,d\theta^2 \tag{3.11}$$

Here the abbreviations

$$\bar{\rho}^2\equiv r^2+\frac{J(J+1)\hbar}{M^2c^2}\cos^2\theta \tag{3.12}$$

and

$$\Delta\equiv r^2-\frac{2MrG_s}{c^2}+\frac{J(J+1)\hbar^2}{M^2c^2}+\frac{\alpha\hbar G_s}{c^3}\mathrm{Tr}\,Q^2 \tag{3.13}$$

have been used, whereas J_3, J, and Q denote the *quantum numbers* of the spin projection, the total spin, and the generalized charge, respectively. {For the Kerr–Newman solution (3.11) to be valid the *classical* assignment $J_3=[J(J+1)]^{1/2}$ has to be used.}

4. PARTICLE INTERPRETATION IN COLOR GEOMETRODYNAMICS

In consideration of the back-coupling of the spinor fields ψ to the generalized Einstein equations (2.21) it is extremely difficult to solve the Heisenberg–Pauli–Weyl equation (2.23) exactly. Therefore, as a first step it is justified to discuss the effect of the nonlinear coupling of the spinors ψ via the generalized spin–internal-spin density S^ν in a *fixed* background metric of strong gravity as given in the preceding Section.

The account of CGMD on a possible *unification* of all particle forces will be analysed by assimilating the scheme A of the Yang–Mills unification of strong, weak and electromagnetic interactions proposed by Pati and Salam (1973) although CGMD corresponds rather (see 2.1) to a $[SU(4)\otimes]^4$-

unification (Elias et al., 1978) of basic particle forces. In the latter, *fractional* charges can be equally well assigned to the quark fields. In both schemes the array (2.7) of the fundamental spinors can be identified with hypothetical hadronic constituents and the known leptons as follows:

$$\psi=\left|\begin{array}{cccc|c} \mathcal{P}_a & \mathcal{P}_b & \mathcal{P}_c & \nu_e & \text{up} \\ \mathcal{N}_a & \mathcal{N}_b & \mathcal{N}_c & e^- & \text{down} \\ \lambda_a & \lambda_b & \lambda_c & \mu^- & \text{strange} \\ \hline c_a & c_b & c_c & \nu_\mu & \text{charm} \\ \text{red} & \text{yellow} & \text{blue} & \text{lilac} & \end{array}\right. \quad \updownarrow \text{flavors} \qquad (4.1)$$

$$\longleftarrow \text{(body)colors} \longrightarrow$$

Because of the nonlinear term in (2.23), each of these 16 spinorial "constituents" will "feel" a self-interaction and furthermore an "external potential" produced by all other *massive* fermions.

The following discussion will frequently refer to a related *scalar* theory in order to simplify some arguments. In order to do so, it may be assumed that the 4×4 complex scalar fields

$$\varphi\equiv\{\varphi^{(f,c)}(x)|f,c=1,\ldots,4\} \tag{4.2}$$

obey the nonlinear Heisenberg–Klein–Gordon equation

$$\left\{\Box-\frac{3\epsilon\mu c}{8\hbar}l^2|\varphi|^2+\frac{9}{256}l^4|\varphi|^4+\left(\frac{\mu c}{\hbar}\right)^2\right\}\varphi=0 \tag{4.3}$$

in curved space-time. Then this theory is founded on a dynamics that is similar to those derived for the spinor case, the reason being, that the HKG equation (4.3) may be related (but not identically) to the "squared" version of (2.23) (see Deppert and Mielke, 1979). Furthermore, (4.3) belongs to the few scalar equations known to admit *stable* (Anderson, 1971) spherically symmetric solutions in a flat space-time. (In this semi-classical approach, the nonrenormalizability of a $|\varphi|^6$ self-interaction may be disregarded.)

4.1. Leptons as Goldstone Spinors? Taking $\mu^2<0$ [in (4.3)] it may be supposed that the (unphysically) large $U(4)$-color symmetry is *spontaneously broken* by the ground state, in such a way that the *scalar* fields $\varphi^{(f,4)}$ associated with leptons remain massless (at least with respect to strong interactions, i.e., their mass being of purely electromagnetic origin), whereas the "quark"-type scalar fields pick up real masses of say $\mu\sim300$ MeV. Presumably, this will work out similarly as in Goldstone's model field theory (see, e.g., Taylor, 1976). If these arguments could be carried over to the nonlinear spinor theory given by (2.23), leptons would be the related massless spinors of CGMD. (In a supersymmetric model (Capper et al., 1976) only the muonic neutrino ν_μ could perhaps be viewed as a "*Goldstone spinor*"). This idea gets further support from the fact that the spin of massless spinors is dual to a lightlike axial vector. Associated with the latter is the torsional part of the gauge connection B_μ, which in the massless case may then be removed by an appropriate gauge transformation. Therefore, in the treatment of leptons the nonlinear self-interactions can be neglected to a certain extent as is usually the case in quantum electrodynamics (QED). The fact that the coupling given by the torsion in the Dirac–Weyl equation (2.23) is effective only for baryons but not for (massless) leptons is already made use of in a model by Finkelstein (1961). In his theory, the torsion of the space-time originates from Yukawa-type pseudoscalar fields.

On the other hand, the consideration of Cartan's torsional self-coupling at a length scale of the order of the Fermi length

$$l_F=(G_F/\hbar c)^{1/2}=6.737\times 10^{-17}\text{ cm} \tag{4.4}$$

may provide an alternative model of *weak interactions* (Hehl, 1970; De Sabbata and Gasperini, 1978).

4.2. Baryons as Black Solitons? Opposite effects occur in the case of massive spinors. As was suggested in the "new geometrodynamical model of baryons" (Mielke, 1977d), the nonlinear terms are then expected to be essential for the formation of *bound states* of three quarks (e.g., $\mathcal{P}$, $\mathcal{N}$, and λ forming a Λ particle) such that the baryons represented by these states are in a *color singlet*. (The additional charm symmetry is assumed to be badly broken.) As a first step consider the Heisenberg–Klein–Gordon equation (4.3) corresponding to (2.23), which is given in a stationary background. Expanding the stationary solution into spherical harmonics $Y_\iota^c(\theta,\phi)$ reveals that a trivial and also a *nontrivial* bound state of n "soliton-type" solutions occur for "angular momentum" $\iota=0$ and $\iota=(n-1)/2$ (Deppert and Mielke, 1979). If these solutions are sufficiently *localized* the account of the quark fields ψ (or φ) to the f-gravity background can be neglected and the complete field equations reduce to the Einstein–Maxwell-type system (3.4) far away from the center of the bound state. These equations admit (quite uniquely) the Kerr–Newman (–de Sitter) solution (3.11) in the stationary case. This background produces an *effective curvature potential* (Brill et al., 1972) for scalar waves φ which reduces in the Schwarzschild–de Sitter case to

$$V^{\iota}{}_{\substack{\text{Mink.}\\ \text{(Eucl.)}}}(\rho^*)=\left(1\,(\overset{-}{+})\,\frac{1}{\rho}-\frac{1}{3}\frac{l^{*2}}{2\pi}\Lambda\rho^2\right)\left(\frac{\iota(\iota+1)}{\rho^2}\,(\overset{+}{-})\,\frac{1}{\rho^3}-\frac{2}{3}\frac{l^{*2}}{2\pi}\Lambda\right) \tag{4.5}$$

By introducing the dimensionless radial coordinate

$$\rho=\frac{M^*c}{2\hbar}r \tag{4.6}$$

the potential (4.5) is implicitly expressed in terms of Wheeler's (MTW, p. 663) "tortoise" coordinate

$$\rho^* \equiv \int d\rho\, \rho \left(\rho\,(\mp)\, 1 - \frac{1}{3}\,\frac{l^{*2}}{2\pi}\,\Lambda \rho^3 \right)^{-1} \tag{4.7}$$

De Sitter "microuniverses" could provide particularly useful models of the hadronic background space-time [see Mielke (1977c) for a review of field theory in de Sitter space]: In the *anti*-de Sitter case ($M=0$, $\Lambda=-3/R^2$) exhibiting an $O(2,3)$ symmetry, $V^0_{\text{Mink}}(\rho^*)$ generates a completely confining "bag" for hadrons of the *harmonic oscillator* type (Salam and Strathdee, 1978; Caldirola et al., 1978), whereas the finite barrier of the Schwarzschild case ($\Lambda=0$) may be operative for a more realistic partial confinement (Figure 1).

A concept of describing *extended* particles by means of *strong* internal curvature has already been suggested by Lanczos (1957). If this internal space is assumed to be an Einstein space $R_{\mu\nu}=\Lambda f_{\mu\nu}$, it can be shown

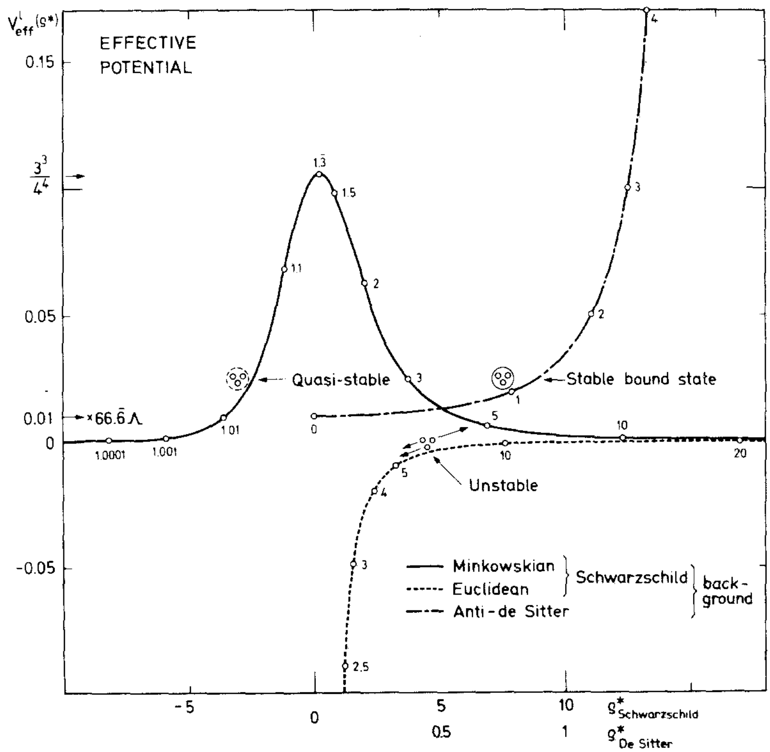

Fig. 1. Confining potentials in strong gravity.

(Komar, 1964; Vigier, 1966) that the phenomenological particle symmetries like $SU(3)$ or $SU(2, 1)$ emerge rather naturally. A geometrical derivation of these internal symmetry groups, however, is fundamentally related to the difficult problem of embedding the internal manifold in the external space-time (Ne'eman, 1965; Salam and Strathdee, 1978).

The nontrivial Ansatz (Deppert and Mielke, 1979) admitted by the HKG equation (4.3) in a Schwarzschild background indicates that even for *color excited* $I_c = \iota = 1$ "quarks" the resulting bound state appears essentially as *black* far away from its center (provided that the quarks carry the *body* colors red, yellow, and blue). According to the general view adopted in CGMD, baryons are represented by *black solitons* [or *geons* (Wheeler, 1962)]. This terminology refers also to the *black-hole-type* background of the confining *tensor gluon* field which together with the torsional nonlinearity in the wave equation is the reason that the color of the "solitonic" bound states may become *transcendent* (Wheeler, 1971a; Bekenstein, 1972; Pati et al., 1975) (or *black*) far away from its center. This renders the ad hoc *assumption* of conventional color models (Greenberg and Nelson, 1977) that physical particles occur only in color singlets unnecessary. Briefly stated, there is "*color without color*" in CGMD!

4.3. Black Soliton Mass Formula. In general relativity, the total mass M of a *black hole* as measured at spatial infinity is given by the formula derived by Christodoulou and Ruffini (1971) for the Kerr–Newman solution. With regard to the background (3.11) it reads

$$\frac{M^2}{M_{\text{ir}}^2} = \left(1 + \frac{\alpha \hbar c}{4 M_{\text{ir}}{}^2 G_S} \operatorname{Tr} Q^2\right)^2 + \frac{J(J+1)\hbar^2 c^2}{4 M_{\text{ir}}{}^4 G_s^2} \tag{4.8}$$

In accordance with Wheeler's (1974) conjecture which states that "a black hole has no hair" the only adjustable *classical* parameters are its total charge, its total angular momentum, and its *irreducible mass*. The latter is a nondecreasing parameter which is determined by the surface area S_{ar} of the hole via

$$M_{\text{ir}} \equiv \left(\frac{S_{\text{ar}}}{16\pi}\right)^{1/2} \frac{c^2}{G_s} \tag{4.9}$$

Therefore, it can be viewed as of purely *geometrical origin*. In GMD it is natural to identify M_{ir} with the Planck mass M^* of strong gravity as given by equation (1.1). Since the preceding discussion supports the expectation that only the flavor but *not* the color degrees of freedom of the bound state are excited in CGMD, the generalized *Gell-Mann–Nishijima relation*

$$Q = I_3 + \tfrac{1}{2} Y - \tfrac{2}{3} C \tag{4.10}$$

can be assumed in a model with integer charged constituents (Pati and Salam, 1973). For the octet representation within the 20-dimensional representation (2, 1, 0, 0) of $U(4)$ the relation (4.10) leads to

$$\operatorname{Tr} Q^2 = -\frac{4\beta}{\alpha} Y + I(I+1) - \frac{1}{4} Y^2 \tag{4.11}$$

whereas other more complicated cases have been explicitly dealt with by Okubo (1975).

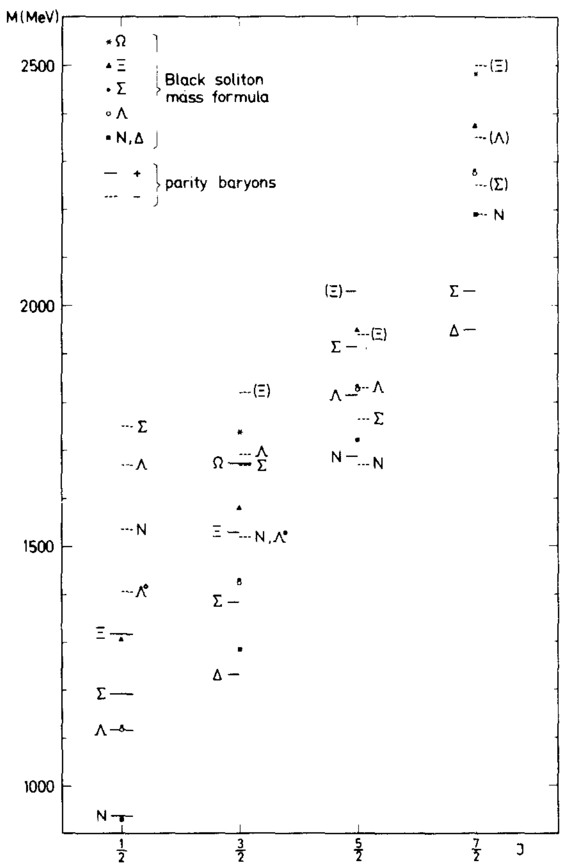

Fig. 2. Black soliton mass formula [M^* fitted for $\Lambda(1115.6)$].

Then the *Gell-Mann-Okubo* type *mass formula*

$$\frac{M^2}{M^{*2}}=\left\{1-\beta Y+\frac{\alpha}{4}\left[I(I+1)-\frac{1}{4}Y^2\right]\right\}^2+\frac{1}{4}J(J+1) \tag{4.12}$$

for *extended* baryons is the result (Mielke, 1977d). Salam (1973) as well as Sivaram and Sinha (1977) made a related suggestion which, however, did not culminate in a specific formula. It generalizes Wheeler's "no hair" conjecture to the $GL(8,\mathbb{C})$ case with the effect that now the quantum numbers of isospin I, hypercharge Y, charm C, and total angular momentum J are the solely disposable characteristics of a black soliton (See also Bekenstein, 1975). In order to obtain a nonzero β the flavor part of G-gauge symmetry needs to be broken down further similarly as in the "eightfold way" scheme (Gell-Mann and Ne'eman, 1964). Using the phenomenological value $\beta=1/5$, the "*black soliton*" *mass formula* (4.12) fits reasonably well (Figure 2) with part of the baryon spectrum as will be discussed in more detail elsewhere (Mielke, 1979b).

4.4 Mesons without Quarks? In conventional models (Kokkedee, 1969), hadronic mesons are interpreted as quark–antiquark pairs $q\bar{q}$ which are bound together by the vector gluons $A_\mu^{(c)}$ of the color gauge groups $U(4)_L^c \otimes U(4)_R^c$. In "quantum chromodynamics" the color electric field is expected to depend as

$$E_b^{(c)} \equiv F_{b0}^{(c)} \sim e^{-|z|\mu_\pi c/\hbar} \tag{4.13}$$

on the distance z from the symmetry axis (see, e.g., 't Hooft, 1977). Since the force between two quarks would then be independent of their spatial separation, a color confinement mechanism for nonsinglet states would have been achieved which qualitatively would resemble strings.

Contrary to this, in CGMD a purely geometrical picture of mesons may emerge as follows.

(a) *Pseudoscalar* and *vector mesons* consist only of gluon field (and flavored flux) lines, which, however, are *trapped* in the *wormhole topology* $R\times S^1\times S^2$ of the underlying space-time of hadronic dimensions. Then, similarly as in Wheeler's wormhole model (Wheeler, 1962, 1978), the mouths of a handle apparently are the origins of a quark–antiquark pair which seemingly are connected by a gluon string (Figure 3). This view is consistent with the claim that a *scattering process* (Mielke, 1977a) of black solitons representing baryons should produce such color-confining topological *Einstein–Rosen bridges* (Einstein and Rosen, 1935) in space-time. As these may carry away angular momentum corresponding to $J=0,1$, it would be desirable to achieve a generalization to *colored Kerr–Newman wormholes*. However, such a global extension of (3.11) is intricate and can lead to causality violations (Carter, 1968). In a related view (Drell, 1978) it has been suspected that hadronic mesons are rather like magnetic dipoles.

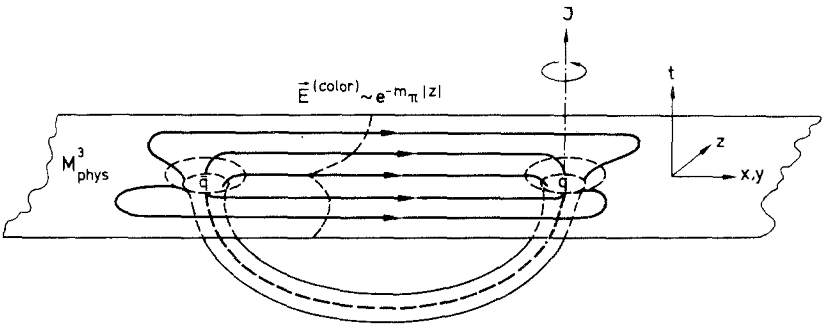

Fig. 3. Hadronic meson regarded as a colored Kerr–Newman wormhole.

(b) *Tensor Mesons*. As the two-tensor theory of Isham et al. (1971, 1974) was originally devised to explain the observed massive nonet of the strongly interacting spin-2^+ mesons f, f', A_2 and $K^*(1430)$, its extension to color is straightforward. Thereby one only adds the colored tensor gluons necessary for the quark confinement in baryons, as already discussed. However, it is important to point out that the physical f-mesons may be more appropriately represented by *nonlinear* f-gravitons (in the sense of Penrose, 1976), in particular with regard to the issue of *quantizing* gravity (Isham et al., 1975). This has to be contrasted with pertubative approaches in a linearized theory.

Finally the *saturation problem* of the conventional quark model (Kokkedee, 1969) may have a surprising solution in CGMD: Mesonic hadrons are not interpreted as quasistable $q\bar{q}$ bound states but are rather explained by color- and flavor-carrying gauge fields trapped in the multiconnectedness of space-time. Only baryons consist of "real" quark-type fundamental spinor fields, which, however, are confined by the curvature potentials of strong gravity. If the gauge subgroup $U(3)^c$ is, as it is usually assumed, an *exact* symmetry, due to the *torsional* self-coupling in the generalized Heisenberg–Pauli–Weyl spinor equation *exactly three* quarks should form a bound state.

5. REGAINING OTHER BAG MODELS FROM CGMD

For future developments it is instructive to point out some interrelations of the geometrodynamical confinement scheme with corresponding mechanisms proposed in *phenomenological bag models* (see Hasenfratz and Kuti, 1978, for a review): In the nonlocal theory of the MIT group (Chodos et al., 1974) the term $-|f|^{1/2}B$ is added to the Lagrangian of the matter fields, where B is a positive potential energy per unit volume. In order to compress the "bag" against the outward pressure of an effectively massless quark gas, B is nonzero only for that region of space that contains hadron fields. This assumption is also necessary in order to confine the gluon vector fields $A_\mu^{(c)}$.

In CGMD the volume tension B corresponds to a nonzero cosmological constant

$$\Lambda(c) = -\tfrac{1}{2} l^2 B(c) \tag{5.1}$$

of the hadronic "minicosmos." According to Salam and Strathdee (1978) a $SU(2)^c \otimes SL(2,\mathbb{C})^c$ gauge theory may yield a *color-sensitive* $\Lambda(c)$ such that in an anti-de Sitter-type confining background color singlets are produced which in their turn do not generate a nonzero $\Lambda(c=0)$.

Therefore, the inclusion of the so-called cosmological term in CGMD may not at all be the "biggest blunder of my (Einstein's) life" (citation from MTW, p. 410), at least not with respect to particle physics.

As shown by Creutz and Soh (1975), the MIT bag model can be obtained from a local field theory in a strong-coupling limit. Their analysis also exemplifies a connection with the SLAC bag model (Bardeen et al., 1975) in which the quark field ψ interacts with a neutral scalar field σ the dynamics of which is determined by the Lagrangian density

$$\mathcal{L}_\sigma = \tfrac{1}{2}(\partial_\mu \sigma)(\partial^\mu \sigma) - H\sigma^4 + 2Hf^2\sigma^2 \tag{5.2}$$

In both models the quartic self-interaction is an essential ingredient in order to produce the bag. It is therefore important to note that such a confining scalar potential already is inherent in CGMD as a special case. To see this, apply the conformal change

$$f^{\mu\nu} \to \bar{f}^{\mu\nu} = \sigma^{-4/(n-2)} f^{\mu\nu} \tag{5.3}$$

of the strong gravity metric to the Einstein Lagrangian density (2.20) to which $\mathcal{L}_{\mathrm{GMD}}$ reduces in the case of vanishing spinor fields. In a space-time having n dimensions this procedure yields

$$\mathcal{L}_E(\bar{f}^{\mu\nu}) = |\det f^{\mu\nu}|^{-1/2}\{\sigma^{(n+2)/(n-2)}\bar{R}\sigma - 2\Lambda\sigma^{2n/(n-2)}\} \tag{5.4}$$

Since with respect to (5.3) the scalar curvature $\bar{R}$ transforms as (Mielke, 1977b, Appendix, equation A.6)

$$\sigma^{(n+2)/(n-2)}\bar{R} = R\sigma - \frac{4(n-1)}{n-2}\Box\sigma \tag{5.5}$$

up to a total divergence

$$\mathfrak{L}_{\mathrm{GMD}} = |\det f^{\mu\nu}|^{-1/2}\left\{ \frac{4(n-1)}{n-2} f^{\mu\nu}(\partial_\mu\sigma)(\partial_\nu\sigma) - 2\Lambda\sigma^{2n/(n-2)} + R\sigma^2 \right\} \quad (5.6)$$

is the result. This is proportional to (5.2) if the hadronic metric of the four-dimensional space-time is *conformally flat* and the values of the cosmological constant and of the scalar curvature are related by

$$\Lambda = 6H, \qquad R = 24Hf^2 \quad (5.7)$$

respectively.

6. QUARK CONFINEMENT FOUNDED ON THE STRUCTURE OF SPACE-TIME?

If the confinement mechanism provided by CGMD is really *fundamental* it should crucially depend on the signature and on the dimension n of the space-time manifold.

Since the Riemann curvature tensor is completely determined by the Ricci tensor if $n \leqslant 3$, Einstein's *vacuum* equations (2.24) with $\Lambda(c) = 0$ *outside* the hadron admit *nonflat* confining solutions only if $n \geqslant 4$, a result which is independent of the signature of the manifold.

Consider now for $n=4$ a Schwarzschild-type hadronic background defined in a locally Euclidean "space-time." Since the corresponding Newtonian potential is related to the metric coefficients by (MTW, p. 449)

$$\Phi_N = \frac{1}{2}(g_{00} - \eta_{00}) \sim -\frac{M^* G_s}{r} \quad (6.1)$$

the line element

$$ds^2 = c^2\left(1 + \frac{2M^* G_s}{rc^2}\right)dt^2 + \left(1 + \frac{2M^* G_s}{rc^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \quad (6.2)$$

has to be considered as the "Euclideanization" of (3.11) for $J_3 = J = Q = 0$. As expected intuitively in this case there does not exist anything that would resemble a horizon (at $r = 2MG_S/c^2$). This is reflected in the fact that the corresponding effective potential (4.5) will not exhibit any barrier (see Figure 1) that could prevent an accumulation of quarks having positive energy from disintegrating into its constituents.

This has to be opposed to the case of a curved manifold which is *asymmetrically broken up* into one with three spacelike and one timelike dimensions. Then the curvature barrier of $V^{\iota}_{\text{Mink}}(\rho^*)$ becomes operative for a (partial) confinement of classical nonlinear waves. As discussed in Section 4.2 a complete quark confinement can be achieved by the tensor forces in anti-de Sitter "microuniverses" (Figure 1). If this view is respected by nature, baryons would resemble mirror images of the whole universe, as is anticipated in Leibniz' theory of *monads* (Leibniz, 1714; see also Wheeler, 1971b). In conclusion the following *deep truth* (but not according to Niels Bohr's terminology) may be suspected as the outcome of a new geometrodynamical model of particles:

The reason for the nonexistence of free quarks is that we live in a **four**-*dimensional, locally* **Minkowskian** *space-time.*

If nature had chosen a space-time with different characteristics, the proton, e.g., in this conceptual framework would be too unstable for physicists to come into being (compare Salam, 1977).

This concluding remark is very much in the spirit of a "*Darwinistic*" explanation of the fundamental physical constants discussed by MTW (p. 1216).

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